Angular Adaptivity for Radiative Transfer Equation Using the Finite Volume Method

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Outline

Radiative Heattransfer

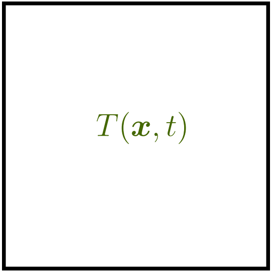
Finite Volume Method

Angular Adaptivity

Status

Next Steps

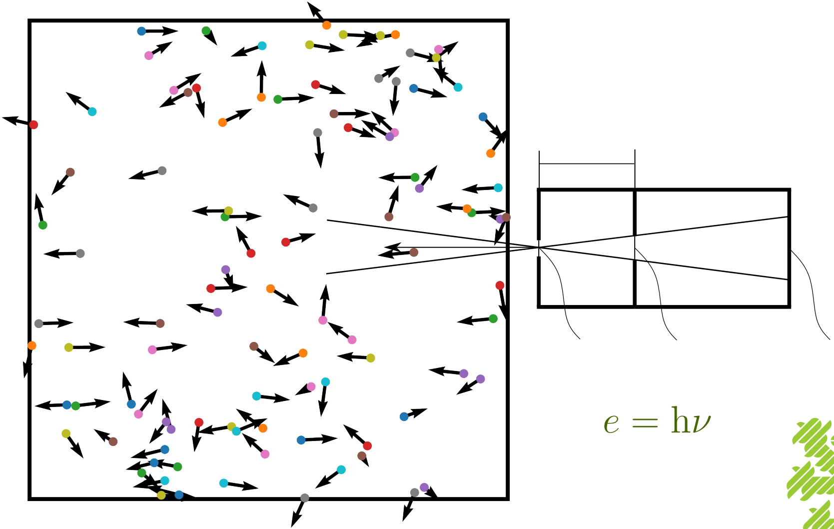
Radiative heattransfer in a nutshell



$$\frac{\partial \rho h}{\partial t} + \nabla \cdot (\rho h \boldsymbol{v}) + \nabla \cdot \boldsymbol{q} = \boldsymbol{\sigma} : (\nabla \boldsymbol{v}) + q_{\text{chem}} + q_{\text{rad}}$$

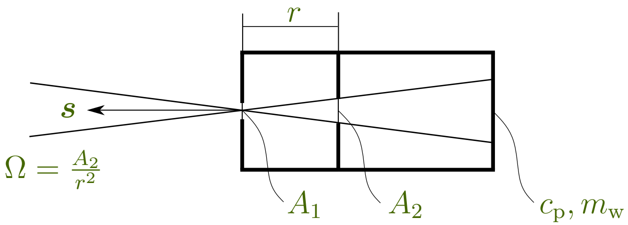
$$q_{\text{rad}} = \int_{\lambda} \int_{4\pi} I_{\lambda}(\boldsymbol{s}) \mathrm{d}\Omega \mathrm{d}\lambda$$

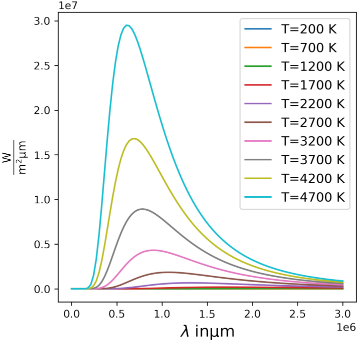
Radiative heattransfer in a nutshell



$$e = \mathrm{h}\nu$$

Radiative heattransfer in a nutshell





$$m_w c_{p,w} \frac{\Delta T}{\Delta t} = P(\boldsymbol{s}, \boldsymbol{x})$$

$$\Omega = \frac{A_2}{r^2}$$

$$\frac{m_w c_{p,w}}{A_1 \Omega} \frac{\Delta T}{\Delta t} = \frac{P(\boldsymbol{s}, \boldsymbol{x})}{A_1 \Omega} = I_\lambda(\boldsymbol{s}, \boldsymbol{x})$$

Radiative heattransfer in a nutshell (RTE)

$$\frac{1}{c}\frac{\partial I_\lambda}{\partial t} + \boldsymbol{s} \cdot \nabla I_\lambda(\boldsymbol{s}, \boldsymbol{x}) = \underbrace{-\kappa_\lambda I_\lambda}_{\text{Absorbtion}} + \underbrace{\kappa_\lambda I_\text{b}}_{\text{Emission}} - \underbrace{\sigma_{\boldsymbol{s}} I_\lambda}_{\text{out}-\text{Scattering}} + \underbrace{\frac{\sigma_\text{s}}{4\pi}\int_{4\pi} I_\lambda \Phi(\boldsymbol{s}', \boldsymbol{s}) \mathrm{d}\Omega'}_{\text{in}-\text{Scattering}}$$

- restrict to on band $\rightarrow$ skip λ
- neglect scattering

$$\boldsymbol{s} \cdot \nabla I = -\kappa I + \kappa I_\text{b}$$

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Finite Volume Method

$$\boldsymbol{s} \cdot \nabla I = -\kappa I + \kappa I_{\mathrm{b}}$$

$$\int_{\Omega^{d_c}} \int_{V_c} \nabla \cdot (\boldsymbol{s} I(\boldsymbol{s}, \boldsymbol{x}))\, \mathrm{d}\Omega \mathrm{d}V = \int_{\Omega^{d_c}} \int_{V_c} -\kappa I + \kappa I_{\mathrm{b}}\, \mathrm{d}\Omega \mathrm{d}V$$

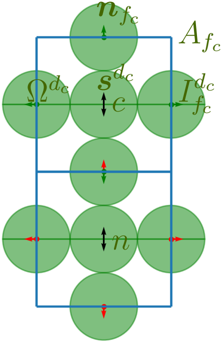
$$\int_{\Omega^{d_c}} \int_{A_{f_c}} I(\boldsymbol{s}, \boldsymbol{x}) \boldsymbol{s} \cdot \boldsymbol{n}\, \mathrm{d}\Omega \mathrm{d}V = \int_{\Omega^{d_c}} \int_{V_c} -\kappa I + \kappa I_{\mathrm{b}}\, \mathrm{d}\Omega \mathrm{d}V$$

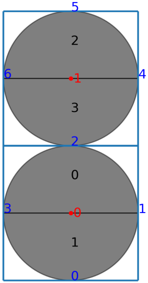
$$\sum_{f_c=1}^{F_c} I_{f_c}^{d_c} \boldsymbol{s}^{d_c} \cdot \boldsymbol{n}_{f_c} A_{f_c} = -\kappa_c I_c^{d_c} \Omega^{d_c} V_c + \kappa_c I_{\mathrm{b},c} \Omega^{d_c} V_c$$

$$\boldsymbol{s}^{d_c} = \int_{\Omega^{d_c}} \boldsymbol{s} \mathrm{d}\Omega$$

Finite Volume Method

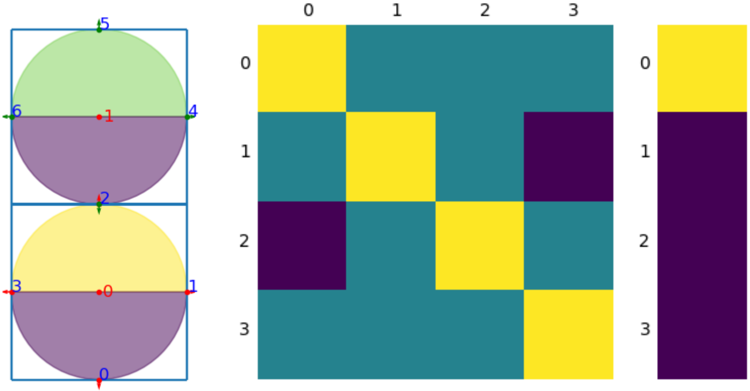
$$\sum_{f_c=1}^{F_c} I_{f_c}^{d_c} \boldsymbol{s}^{d_c} \cdot \boldsymbol{n}_{f_c} A_{f_c} = -\kappa_c I_c^{d_c} \Omega^{d_c} V_c + \kappa_c I_{\mathrm{b},c} \Omega^{d_c} V_c$$



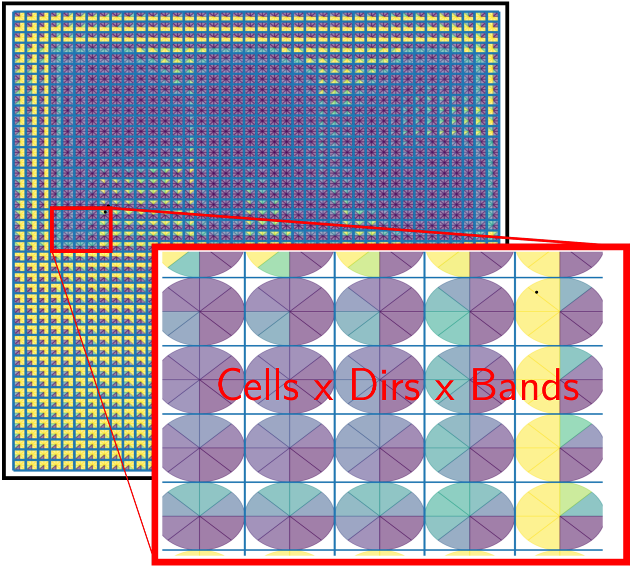


$$I_{f_c}^{d_c} = \alpha_{f_c}^{d_c} I_c^{d_c} + (1 - \alpha_{f_c}^{d_c}) I_n^{d_c}, \quad \alpha_{f_c}^{d_c} = \begin{cases} 1, \text{if } \boldsymbol{s}^{d_c} \cdot \boldsymbol{n}_{f_c} > 0 \\ 0, \text{if } \boldsymbol{s}^{d_c} \cdot \boldsymbol{n}_{f_c} < 0 \end{cases}$$

Matrix



Example (Nerd-Art)



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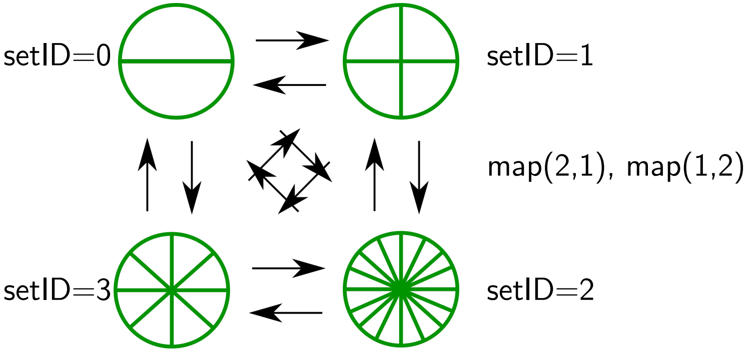
Mapping

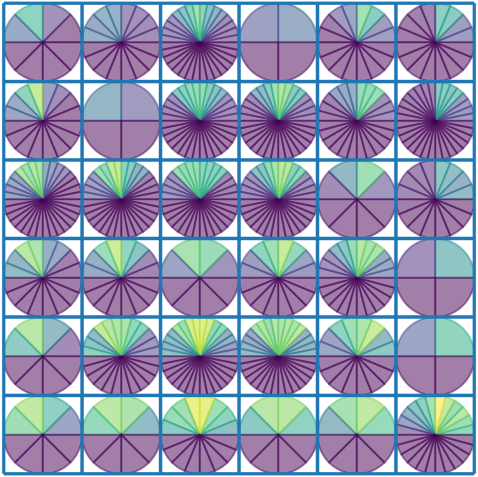


$$\sum_{f_c=1}^{F_c} \alpha_{f_c}^{d_c} I_c^{d_c} \boldsymbol{s}^{d_c} \cdot \boldsymbol{n}_{f_c} A_{f_c}$$

$$+ \sum_{f_c=1}^{F_c} \sum_{d_n=1}^{D_n} (1-\alpha_{f_c}^{d_c}) I_n^{d_n} \boldsymbol{s}^{d_n} \cdot \boldsymbol{n}_{f_c} A_{f_c} \mathrm{map}(\mathrm{sID}(c,d_n), \mathrm{sID}(n,d_n))$$

$$= -\kappa_c I_c^{d_c} \Omega^{d_c} V_c + \kappa_c I_{\mathrm{b},c} \Omega^{d_c} V_c$$

Adaptivity Example



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- 2d Finite Volume solver implemented
- Arbitrary mesh support (all what comes out of gmsh) implemented
- Possible applications found
- TDD system used
- Plot functions for indices, values, matrices implemented
- 2d adaptivity implemented
- Some discussions with senior people about need

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- Evaluate different coupling conditions
- Implement more advanced schemes
- Vectorize code
- Evaluate useful applications
- Extend to 3d
- Implement into fireFoam/FDS
- Develop strategy for automated resolution control
- Calculate benefit
- Publish

Thank you for your attention!

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Literature

[1] John C. Chai, Jeffrey P. Moder. "RADIATION HEAT TRANSFER CALCULATION USING AN ANGULAR-MULTIBLOCK PROCEDURE". In: *Numerical Heat Transfer, Part B: Fundamentals* 38.1 (July 2000), pp. 1–13. ISSN: 1040-7790, 1521-0626. DOI: 10.1080/10407790050131534. URL: http://www.tandfonline.com/doi/abs/10.1080/10407790050131534 (visited on 06/04/2021).

[2] M. F. Modest. *Radiative heat transfer*. Third Edition. OCLC: ocn813855549. New York: Academic Press, 2013. 882 pp. ISBN: 978-0-12-386944-9.